Kinetic-MHD simulation of nonlinear interaction between Alfvén instabilities and energetic particles

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Highlights

- M3D-C1-K is newly-developed kinetic-MHD code based on M3D-C1, to study the interaction between EPs and MHD activities (Alfvén waves, kink/tearing modes etc).
- The chirping of RSAE due to MHD dissipation is studied using linear MHD and nonlinear particle simulation, showing that the splitted chipring modes have different mode structure including widths and peaks.
- Using nonlinear MHD simulation, it is found that the high-*n* modes generated from RSAE can reduce the saturation level and dissipate the AE.

Overview

1. Introduction to the new M3D-C1-K code
2. Linear MHD + nonlinear kinetic: AE frequency chirping
3. Nonlinear MHD + nonlinear kinetic: AE-ZF interaction
4. Summary

Outline

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Introduction to the new M3D-C1-K code

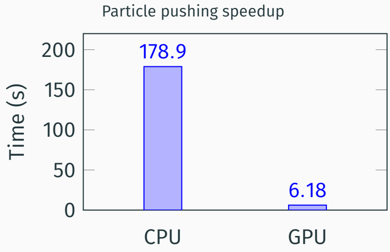
M3D-C1-K is a new hybrid kinetic-MHD code based on M3D-C1, to study the interaction between energetic ions and MHD activities (Alfvén waves, kinetic/tearing modes etc).

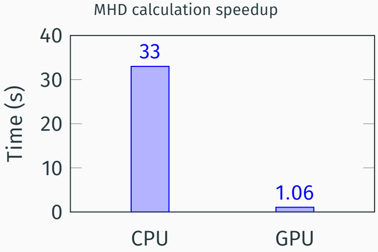
- A new particle pushing scheme based on slow manifold Boris algorithm has been implemented and fully accelerated using GPU.
- Both pressure coupling (like M3D-K) and current coupling (like MEGA) with MHD equations are implemented.
- Several linear benchmarks have been conducted, including fishbone, TAE and RSAE, and good agreements with other codes have been achieved.

C. Liu, S.C. Jardin, H. Qin, J. Xiao, N.M. Ferraro, and J. Breslau, ArXiv:2107.13663 (2021)

GPU acceleration of particle pushing and MHD equation matrix calculation

- Particle pushing is optimized using particle-based parallelization, in which the equation of motions of all particles are calculated parallelly.
- Calculation of matrix used in Galerkin method for MHD equation is optimized by calculating the numerical integral of all the test and basis functions parallelly .
- Significant speedup was obtained using GPU on Traverse cluster at Princeton University.
    - Currently, only the iterative matrix solver has not been optimized for GPUs, which relies on PETSc library.

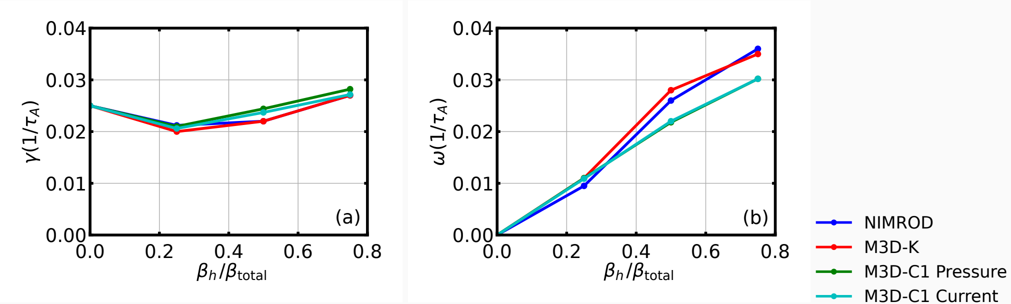




Fishbone simulation result agrees with M3D-K and NIMROD

$$R/a = 2.8, \quad \beta_{total} = 0.08, \quad q_0 = 0.6, \quad q_a = 2.5$$
$$\hat{\rho}_h = v_0/(\Omega_h a) = 0.0125, v_0/v_A = 4$$

Growth rate and real frequency of fishbone from linear simulation

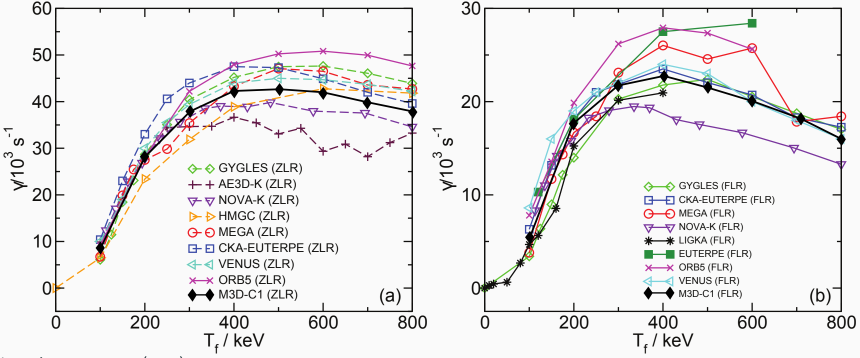


G.Y. Fu, W. Park, H.R. Strauss, J. Breslau, J. Chen, S. Jardin, and L.E. Sugiyama, Phys. Plasmas 13, 052517 (2006).
C.C. Kim and the NIMROD Team, Phys. Plasmas 15, 072507 (2008).

TAE linear simulation without and with FLR effects consistent with previous benchmark

- This is an ITPA collaborative effort to compare different codes and physical model. Several hybrid MHD, gyrokinetic and gyrofluid codes are benchmarked.
  $R/a = 10, \quad \beta \approx 0.2\%, \quad q = 1.71 + 0.16(r/a)^2$
  $n_f = c_3 \exp\left(-\frac{c_2}{c_1} \tanh \frac{\sqrt{s}-0.5}{c_2}\right)$

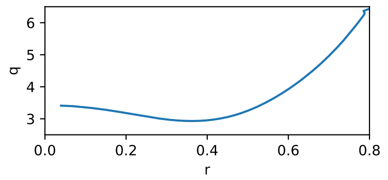
Growth rate from linear TAE simulation without and with FLR

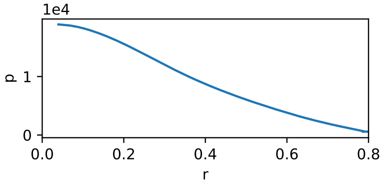


A. Könies, et al., Nucl. Fusion 58, 126027 (2018).
Yawei Hou, et al., Phys. Plasmas 25, 012501 (2018).

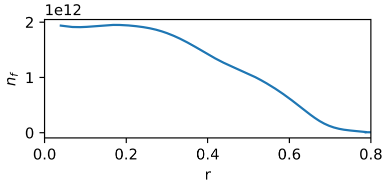
Study of RSAE simulation using DIII-D equilibrium

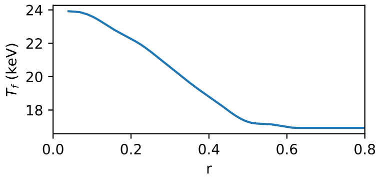
- Several MHD and gyrokinetic codes were employed to study the linear growth of reversed shear Alfvén eigenmode (RSAE) using DIII-D experimental parameters.
- $B_0 = 2\text{T}$, $R = 1.6435\text{m}$, $a = 0.627\text{m}$
- q profile has a minimum at with $q_{min} = 2.93$ at $r = 0.36\text{m}$





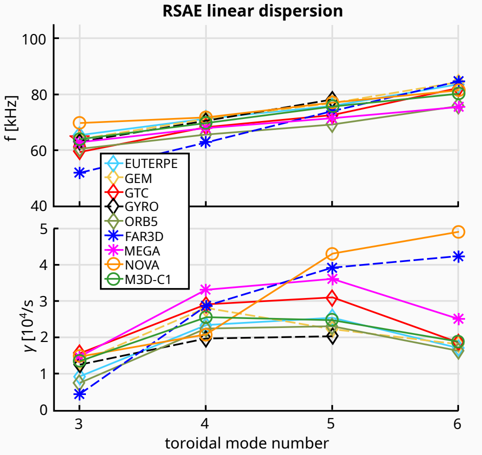
- EP follows a Maxwellian distribution in momentum space.





Linear simulation of RSAE driven in DIII-D tokamak

- Including FLR effects leads to smaller mode growth rate, especially for high-k modes.
- We got almost the same results using pressure coupling or current coupling, meaning that the parallel dynamics are not important.
- Compressional effects ($\delta B_{\parallel}$) are not important.



S. Taimourzadeh et al., Nucl. Fusion 59, 066006 (2019).

Outline

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Frequency chirping simulation with only particle nonlinearity

- Frequency chirping is widely observed in AEs excited by EPs in tokamaks and STs.
- Berk-Breizman theory gives a solid explanation about up and down frequency chirping through clump-hole formation.
- White et al. studied mode frequency chirping using ORBIT, which utilized NOVA to calculate the eigenmode structure, and information from ORBIT particles to calculate mode amplitude and phase changes.
  - This study shows that the frequency chirping is mainly caused by nonlinear effects in particles instead of that in MHD.
- To study the frequency chirping, we do a linear MHD simulation plus a nonlinear particle simulation.
  - Finite values of viscosity and resistivity are set to cause a mode damping rate (γ_d) that drive chirping.
  - The toroidal mode number is limited whereas the mode structure is allowed to evolve.

R.B. White, V.N. Duarte, N.N. Gorelenkov, E.D. Fredrickson, M. Podestà, and H.L. Berk, Phys. Plasmas 26, 092103 (2019)

G. Meng, P. Lauber, Z. Lu, and X. Wang, Nucl. Fusion 60, 056017 (2020)

A. Bierwage, R.B. White, and V.N. Duarte, Plasma and Fusion Research 16, 1403087 (2021)

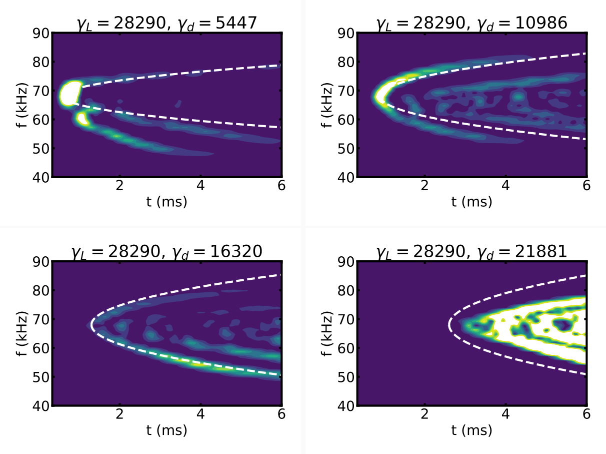
Modes frequency chirping after saturation

- After the mode saturated, it splits into two modes, chirping upward and downward with similar rate. More splittings can happen later.
- The result shows that frequency chirping rate is consistent with the Berk-Breizman theory for small γ_d cases, but not the marginal case ($\gamma_L \approx \gamma_d$)

$$\delta f = \frac{16\sqrt{2}}{\pi^2 3\sqrt{3}} \gamma_L \sqrt{\gamma_d t}$$

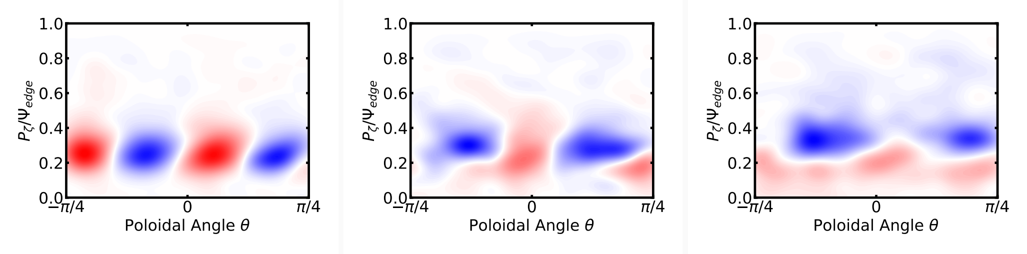
- For small γ_d cases, the chirping modes amplitudes are much smaller compared to initial saturation amplitude.

Spectrogram of RSAE for $r = 0.35$m with different damping rate (γ_d)



Perturbed particle distribution δf shows no clump-hole separation

Evolution of δf during chirping for $\gamma_d = 16320$



- Like the previous results, we observe separation of clumps and holes in f_{EP} in phase space during chirping.

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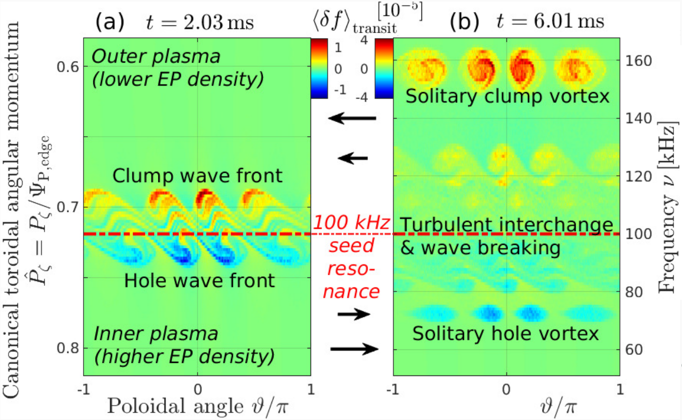

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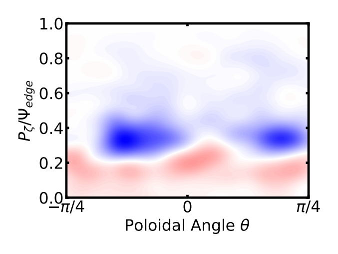
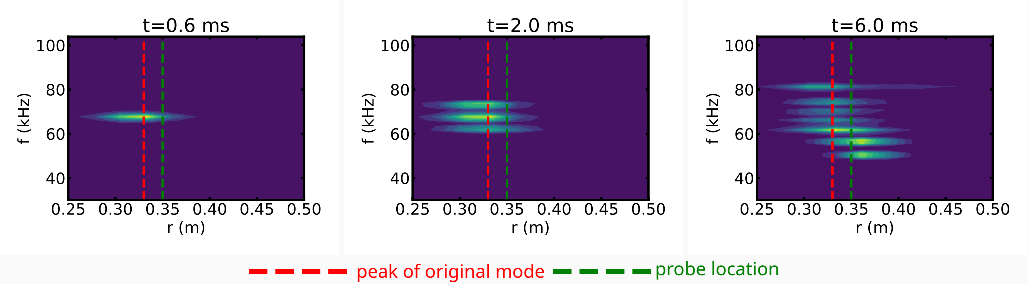


and holes in f_{EP} in phase space

A. Bierwage, R.B. White, and V.N. Duarte, Plasma and Fusion Research 16, 1403087 (2021)

Change of mode structure during chirping

Evolution of mode structure of different chirping modes for $\gamma_d = 16320$

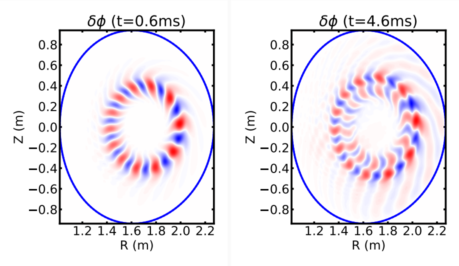


- The mode structure of the modes after splitting are not identical. The low frequency branch shifts outward and is localized, whereas the high frequency branch shifts inward and has a broader radial distribution.
- For a probe at a certain location, the observed chirping can be asymmetric due to the mode structure changes.

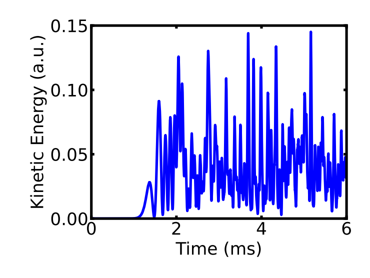
Mode structure splitting and beating of energy

- During the chirping, the two modes are splitted on the radial direction. The up chirping one moves inward while the down chirping one moves outward.
- Given the two modes overlap and have different frequencies, mode structure and the total kinetic and magnetic energies show beating whose frequency is $\omega_1 - \omega_2$.

Mode structure before and after chirping happens



Kinetic energy beating after saturation

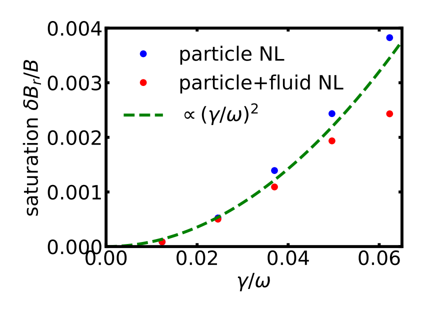


Outline

Nonlinear simulation of excitation and saturation of RSAE

- We did 3D simulation including fluid nonlinear terms and zero MHD dissipation, to study the effect of mode-mode coupling on RSAE saturation.
  - Since $n = 4$ is the fastest growing RSAE, the simulation is limited in $\phi \in [0, 2\pi/4]$, to only study modes with n as multiples of 4 (including $n = 0$).
  - Flat equilibrium pressure and density profiles are used to suppress high-n ballooning modes.
- We find that using MHD simulation without density perturbation, the saturation level is reduced by the fluid nonlinearity, falling below the $\delta B \sim (\gamma/\omega)^2$ scaling law.
  - This is consistent with the result in Chen (2018) doing gyrokinetc simulation.

Saturation amplitude of RSAE without and with fluid NL



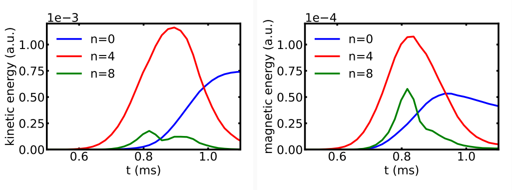
Y. Todo, H.L. Berk, and B.N. Breizman, Nucl. Fusion 52, 094018 (2012).

Y. Chen, G.Y. Fu, C. Collins, S. Taimourzadeh, and S.E. Parker, Physics of Plasmas 25, 032304 (2018).

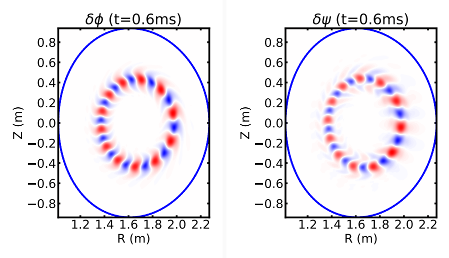
Zonal flow and field generation during the saturation of RSAE

- Both the zonal ($n = 0$) and the higher-n components are excited as a result of the nonlinear mode-mode coupling during RSAE saturation.
    - Growth rate of $n = 0$ and $n = 8$ components are twice the RSAE linear growth rate.
- After RSAE saturation, the AE damps quickly (within 0.2ms) and the mode structured become ZF dominated.

Kinetic and magnetic energy evolution in nonlinear RSAE simulation



Velocity and magnetic field structure before saturation



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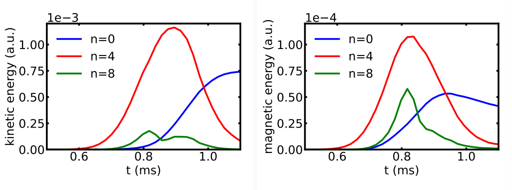
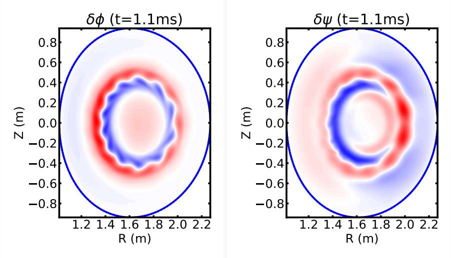


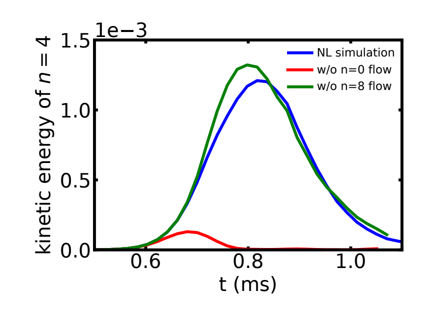
Velocity and magnetic field structure after saturation



Higher-n modes are more important for RSAE dissipation

- By moderating different n components, we find that the higher-n modes play more important roles in dissipating the RSAE.
  - Removing the higher-n component can lead to a larger saturation amplitude.
  - Removing only the zonal flow can significantly reduce the saturation amplitude.
- The zonal flow plays two roles here. It can modulate RSAE, but it can also suppress the growth of higher-n mode thus protect the RSAE from nonlinear dissipation.
  - Without zonal flow, the higher-n modes can grow much faster and dissipate the RSAE quickly.

RSAE kinetic energy evolution with moderated n components



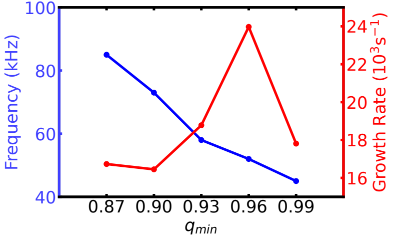
Linear frequencies and growth rates of varying q_{min}

- To study the nonlinear behavior of RSAE during frequency sweeping, we vary the equilibrium q_{min} by changing the toroidal field.
  - Mode frequency decreases as q_{min} increases, which is consistent with linear dispersion relation.

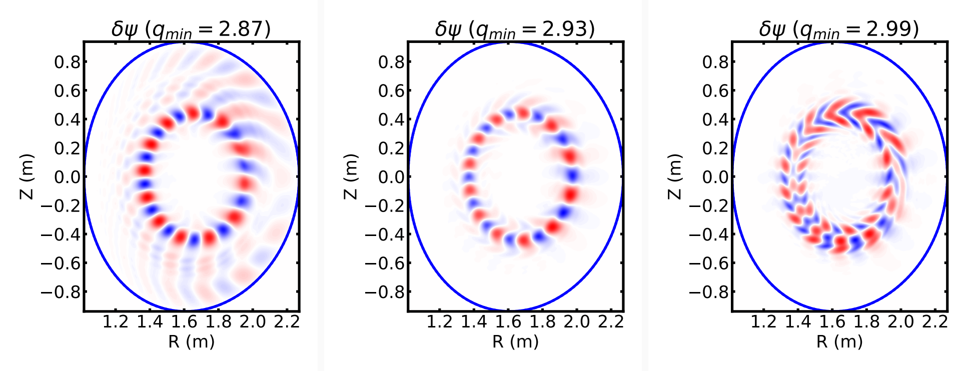
$$f_{RSAE} = \left(\frac{m}{n} - q_{min}\right)\frac{v_A}{2\pi R}$$

  - Linear growth rate peaks at about $q_{min} = 0.96$, which is determined by the balance of EP drive through gradient on radial direction and EP Landau damping.

Frequency and growth rates of RSAE for different q_{min}



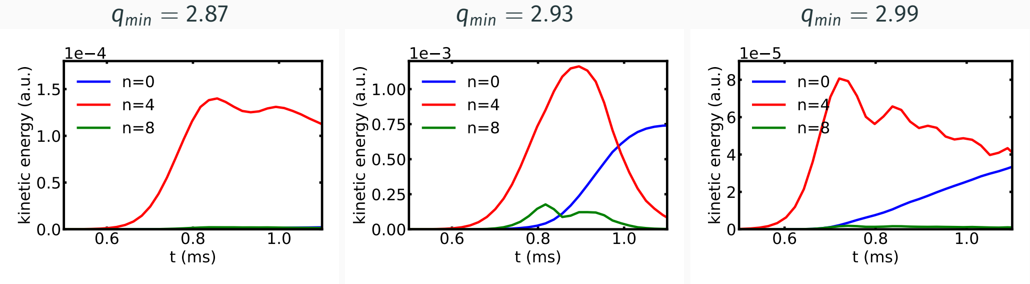
Mode structure of different q_{min}



- For small q_{min}, the mode is little affected by the resonant particles and the mode has a almost constant phase on radial direction.
- For large q_{min}, the kinetic effect from resonant particles can cause strong phase shift along radial direction.

R. Ma, F. Zonca, and L. Chen, Phys. Plasmas 22, 092501 (2015).
Z.X. Lu, X. Wang, P. Lauber, and F. Zonca, Phys. Plasmas 25, 012512 (2018)

Different saturation behavior of RSAEs for varying q_{min}



- For small q_{min}, zonal and higher-n modes are barely generated due to small population of resonant EP, and the mode does not damp after saturation.
- For large q_{min}, zonal and higher-n modes are generated but small given the small RSAE saturation amplitude, and the mode damps slowly after saturation.
- Quick damping happens for $2.93 < q_{min} < 2.96$.

Outline

Summary

- Frequency chirping of RSAEs can be simulated using M3D-C1-K by only including particle nonlinearity, which can save computation time and gives a reasonable result.
    - The beating of mode energy as a result of chirping is observed.
    - The structures of up and down chirping modes are different, which can lead to asymmetric chirping at a certain radial location.
- In nonlinear MHD simulation, the zonal and higher-n fields generated though mode-mode coupling can reduce the AE saturation level and cause quick damping.

Future work:

- Study the frequency chirping with nonlinear MHD effects
- Do the nonlinear simulation in a full torus to study the coupling of multiple modes with different n